SCHOOL OF MATHEMATICS AND PHYSICS

Line integrals of vector fields

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(1) Consider the velocity field $\boldsymbol{v} = x\boldsymbol{i} - y\boldsymbol{j}$. Evaluate the line integral $\int_C \boldsymbol{v} \cdot \boldsymbol{n}\, dS$ directly to calculate the net outward flux from a square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$.

Interpret your results physically knowing that the velocity field is a simple model of fluid flow near a wall ($y \geq 0$).

What can you say about mass conservation?

SciMS online simulations

Click on the links below or type the URLs into your browser's address bar.

- Velocity fields simulation:

  http://teaching.smp.uq.edu.au/scims/Adv_calculus/Velocity_field.html

- Line integrals simulation:

  https://teaching.smp.uq.edu.au/scims/Adv_calculus/Int_vector_field.html

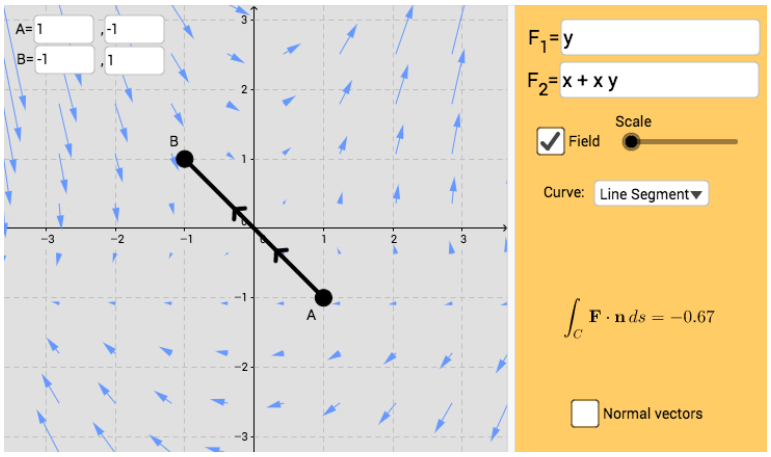

Figure 1: Advanced Calculus→Line integrals of vector fields

(2) Consider the vector field $\boldsymbol{F} = (x - y)\boldsymbol{i} + (x - xy)\boldsymbol{j}$.

a. Use the divergence theorem to explain why the flux out of any circle centred at the line $x = 1$ is zero.

b. Use Stokes theorem to explain why the circulation around any circle centred at the line $y = 2$ is zero.

c. Find the circulation around a circle of unit radius centred at $x = 0$, $y = 1$.

d. Is $\boldsymbol{F}$ is conservative?

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