

## SCHOOL OF MATHEMATICS AND PHYSICS

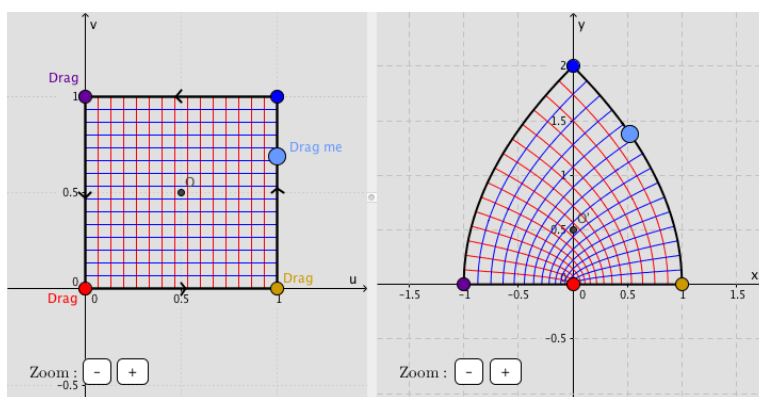
### Non linear transformations in $\mathbb{R}^2$

(1) Explore the transformation defined by the equations

$$x = u^2 - v^2 \quad y = 2uv$$

by using the following simulation: Parabolic transformation. Click on the link below (or type the URL in the address bar of your web browser):

[https://teaching.smp.uq.edu.au/scims/Adv\\_calculus/Nonlineart.html](https://teaching.smp.uq.edu.au/scims/Adv_calculus/Nonlineart.html)



Notice that the image of the square  $A = [0, 1] \times [0, 1]$ , under this transformation, is the region defined between the curves

$$x = 1 - \frac{y^2}{4}, \quad x = \frac{y^2}{4} - 1 \quad y \geq 0$$

Use the simulation to find and analyse the image of the following regions:

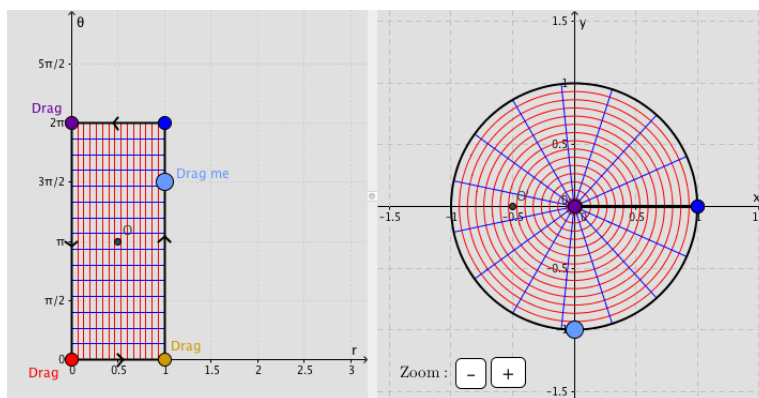
- (a)  $\{(u, v) \mid -2 \leq u \leq 0, 0 \leq v \leq 2\}$
- (b)  $\{(u, v) \mid 1 \leq u \leq 2, 1 \leq v \leq 2\}$
- (c)  $\{(u, v) \mid -1 \leq u \leq 0, -1 \leq v \leq 0\}$
- (d)  $\{(u, v) \mid -1 \leq u \leq 1, -1 \leq v \leq 1\}$

(2) Explore the transformation defined by the equations

$$x = r \cos \theta \quad y = r \sin \theta$$

by using the following simulation: Polar transformation. Click on the link below:

[https://teaching.smp.uq.edu.au/scims/Adv\\_calculus/Nonlineart.html](https://teaching.smp.uq.edu.au/scims/Adv_calculus/Nonlineart.html)



Notice that the image of the rectangle  $A = [0, 1] \times [0, 2\pi]$ , under this transformation, is the unit disk. That is, the set of points  $(x, y) \in \mathbb{R}^2$  such that  $x^2 + y^2 \leq 1$

Use the simulation to find and analyse the image of the following regions:

- (a)  $\{(u, v) \mid 0 \leq u \leq 1, 0 \leq \theta \leq \pi\}$
  - (b)  $\{(u, v) \mid 1 \leq u \leq 2, 0 \leq \theta \leq \pi/2\}$
  - (c)  $\{(u, v) \mid 1 \leq u \leq 2, \pi/2 \leq \theta \leq \pi\}$
  - (d)  $\{(u, v) \mid 1 \leq u \leq 2, \pi/2 \leq \theta \leq 5\pi/2\}$
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