SCHOOL OF MATHEMATICS AND PHYSICS

Fundamental Theorem of Calculus

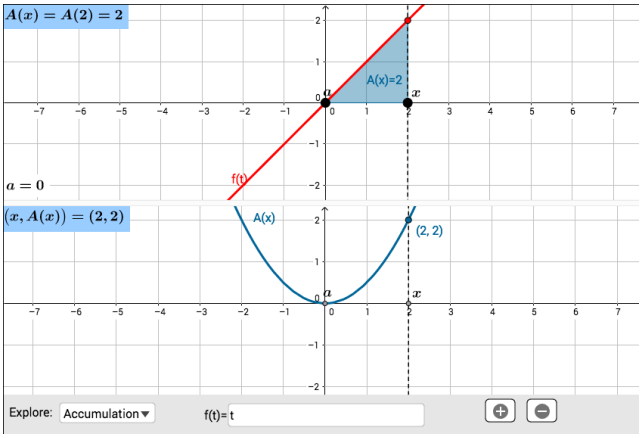
Introduction

In the following simulation, the top graph shows the function $f(t)$ and the value of the definite integral for each upper limit x, with lower limit a. The definite integral is represented with the shaded region between the graph of the function and the x-axis.

The bottom graph shows the **accumulation function**

$$A(x) = \int_a^x f(t)dt$$

for each upper limit x, with lower limit a.



https://teaching.smp.uq.edu.au/scims/Calculus/FTC.html

How to use the simulation

- Select an option, at the bottom, to explore the **Accumulation function** or the **Derivative** of the accumulation function.
- Drag slowly point x along the x-axis in the top graph to observe the relationship between the two graphs.
- Drag the point a to change its value.
- Change the function $f(t)$. **Remark:** f must be continuous on $\mathbb{R}$.
  - Example 1: `sin(t)`
  - Example 2: `t^2-1/2t+1`
  - Example 3: `1/2 abs(t)`

Activity

Consider the accumulation function

$$A(x) = \int_a^x f(t)\,dt$$

where

$$f(t) = \frac{1}{6}t^2 - \frac{1}{3}t - \frac{1}{2} \qquad \text{and} \qquad a = -2.$$

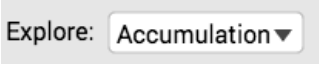
Use the simulation to answer the questions from Part 1 and 2.

Enter the function $f(t)$ as: `1/6t^2-1/3t-1/2`

If needed, drag the point a to change its value.

Part 1 - Analysing the accumulative function

For this part, the option **Accumulation** must be selected:



Drag slowly point x along the x-axis in the top graph to observe the relationship between the two graphs.

1. Approximately, at what value(s) of x does the accumulation function, $A(x)$, have a local maximum? A local minimum? Explain how you know.
2. Drag point x to the x-value at which $A(x)$ has a local maximum. What do you notice about the value of the original function, $f(t)$, at that point?
3. Drag point x to the x-value at which $A(x)$ has a local minimum. What do you notice about the value of the original function, $f(x)$, at that point?
4. Approximately, at what value(s) of x does the accumulation function, $A(x)$, is zero?

Part 2 - Analysing the derivative of the accumulative function

For this part, the option **Derivative** must be selected:

Drag slowly point x in the bottom graph and observe the changes in both graphs.

1. What is the graph on the bottom measuring? How this measurement is related to the top graph?
2. Based on your observations, what is the relationship between the bottom graph $A(x)$ and the original function, $f(x)$?
3. Calculate the formula for the accumulation function using the Fundamental Theorem of Calculus.

Additional question:

Considering the same function $f(t)$, drag now the point a and observe the changes in both graphs. Based on your observations, what is the relationship between the accumulation function $A(x)$ and the constant a?