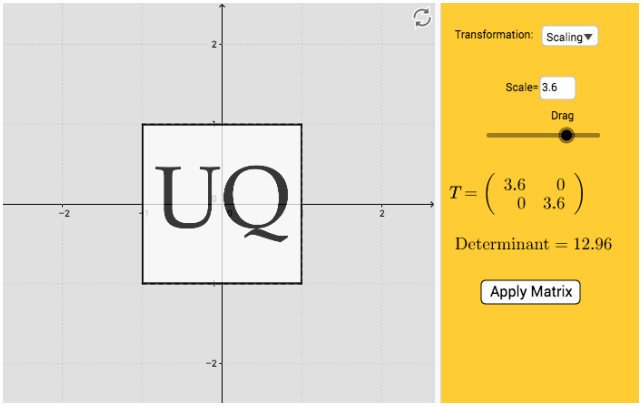
SCHOOL OF MATHEMATICS AND PHYSICS

Operations with matrices

Introduction

Use the following online simulation to help you solve the problems and for checking your answers. Click on the link below or type the URL into your browser's address bar.

https://teaching.smp.uq.edu.au/scims/Linear_algebra/Ltransformation.html



Problems

(1) The determinant of a square matrix is a number that can be related to the area or volume of a region. In particular, the determinant of a matrix reflects how the linear transformation associated with the matrix can scale, rotate, shear or reflect objects.

a. Calculate the determinant of the following square matrices:

$$A_1 = \begin{pmatrix} 5/2 & 0 \\ 0 & 5/2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 1/4 & -1/4 \\ 1/4 & 1/4 \end{pmatrix}, \quad A_6 = \begin{pmatrix} \cos 73^\circ & -\sin 73^\circ \\ \sin 73^\circ & \cos 73^\circ \end{pmatrix} \quad \text{and} \quad A_7 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

b. Describe briefly the transformations of each matrix and its relationship with the determinant.

(2) Consider the matrices

$$A = \begin{pmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \text{ and } \quad C = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

and the square of side 2 centred at the origin. Use the following simulation from the link above to apply each matrix to the square and take note of the determinant.

a. What is the area of the resulting geometric object in each case?

b. Describe a relationship between the area and the determinant of each matrix.